



FITZHUGH–NAGUMO REVISITED: TYPES OF BIFURCATIONS, PERIODICAL FORCING AND STABILITY REGIONS BY A LYAPUNOV FUNCTIONAL

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We study several aspects of FitzHugh–Nagumo’s (FH–N) equations without diffusion. Some global stability results as well as the boundedness of solutions are derived by using a suitably defined Lyapunov functional. We show the existence of both supercritical and subcritical Hopf bifurcations. We demonstrate that the number of all bifurcation diagrams is 8 but that the possible sequential occurrences of bifurcation events is much richer. We present a numerical study of an example exhibiting a series of various bifurcations, including subcritical Hopf bifurcations, homoclinic bifurcations and saddle-node bifurcations of equilibria and of periodic solutions. Finally, we study periodically forced FH–N equations. We prove that phase-locking occurs independently of the magnitude of the periodic forcing.

Keywords: FitzHugh–Nagumo; subcritical and supercritical Hopf bifurcation; homoclinic bifurcation; periodic forcing.

1. Introduction

We consider the FitzHugh–Nagumo (FH–N) equations without diffusion,

$$\begin{aligned}\frac{du}{dt} &= \varepsilon g(u) - w + I, \\ \frac{dw}{dt} &= u - aw,\end{aligned}\tag{1}$$

where $g(u) = u(u - \lambda)(1 - u)$, $0 < \lambda < 1$ and

$a, \varepsilon > 0$. We remark that in the existing literature, the term “FitzHugh–Nagumo system” has been used to refer to models both with and without diffusion.

Although Eqs. (1) have been mentioned in practically every mathematical biology book [Brown & Rothery, 1993; Murray, 1989; Strogatz, 1994], as well as some of their aspects have been studied in different contexts [Armbruster, 1997; Dangelmayr

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