Some midterm-like problems

1) The trace of the parametrized curve $\underset{\sim}{\alpha} : (0, 2\pi) \to \mathbb{R}^2$
$\underset{\sim}{\alpha}(t) := (t + \sin t, 3 - \cos t)$
is called the cycloid. Is $\underset{\sim}{\alpha}$ a regular param. curve? Why or why not?

2) Show that the hyperboloid of one sheet $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad a, b, c \neq 0$

is a regular surface <u>without</u> using any parametrizations.

3) A surface is called "ruled" if it can be described using coordinate charts of the form $(\underset{\sim}{x}, (a,b)\times(c,d))$, where $\underset{\sim}{x}(u,v) := \underset{\sim}{\beta}(u) + v\,\underset{\sim}{\delta}(u)$ for some parametrized curves $\underset{\sim}{\beta} : (a,b) \to \mathbb{R}^3$ and $\delta : (a,b) \to \mathbb{R}^3$.

Show that the hyperboloid of one sheet (see 2) is a ruled surface, using curves $\underset{\sim}{\beta}(u) := (a\cos u, b\sin u, 0)$ and $\underset{\sim}{\delta}(u) := \beta'(u) + (0,0,c)$.

Show that the hyperboloid is a regular surface, using only coord. charts of this kind.

4) If $\alpha: I \to \mathbb{R}^3$ and $\beta: J \to \mathbb{R}^3$ are both curves parametrized by arc length, and trace(α) = trace(β) what can you say about the relationship between the intervals I and J?

5) Show that if α is a curve parametrized by arc length and there is a fixed point $p \in \mathbb{R}^3$ such that every tangent line of α passes through p, then trace(α) is a straight line.

Hint: The tangent line condition is equivalent to $p = \alpha(s) + r(s)\,\alpha'(s)$

for some scalar function r. Differentiate this relation with respect to s and draw some conclusions.

6) Just for fun - we haven't practiced this, so I won't put it on the midterm unless there's strong public demand for it.

The first row is three traces; the second is three curvature plots; the third is three torsion plots. Match the curvatures & torsions to the traces.